ECONOMETRICS

LECTURE NOTES II

FUNCTIONAL FORMS OF REGRESSION MODELS

The term “linear regression” means a regression that is *linear in the parameters* (that is, the parameters are raised to the power of 1 only), *LIP*; it may or may not be *linear in explanatory variables, LIV.*

1. THE LOG-LINEAR MODEL

Suppose we want to estimate the price elasticity of demand, that is, the percentage change in the quantity demanded for a percentage change in the price (of a given commodity). We cannot estimate it from the LIP/LIV model since the slope coefficient of that model simply gives the absolute change in the average quantity demanded for a unit change in the price of the commodity, but this is not elasticity. But it can be readily computed from the so called log-linear model:

$$\ln Y(t)=\alpha+\beta\cdot \ln X(t) \tag{1}$$

(ln=the natural logarithm, that is, logarithm to the base *e*).

Such models are called log-linear (because of linearity in the logs of the variables) or double-log (because both variables are in the log form).

By definition, elasticity is

$$E=\frac{\partial Y}{\partial X}\cdot\frac{X}{Y},$$

where $\partial Y/\partial X$ means the derivative of Y with respect to X.

In the double-log model (1)

$$\beta = \frac{\partial lnY}{\partial lnX} = \frac{\Delta lnY}{\Delta lnX} = \frac{\Delta Y/Y}{\Delta X/X} = \frac{\Delta Y}{\Delta X}\cdot\frac{X}{Y} = E$$

(since *ΔlnY/ΔlnX* is an approximation of *∂lnY/∂lnX* and a change in the log of a number is a relative or proportional change: $\Delta lnY = \Delta Y/Y$, $\Delta lnX = \Delta X/X$).

Thus, if Y represents the quantity of a commodity demanded and X represents its unit price, β measures the price elasticity of demand.

Actually, in general, elasticity is equal to the slope times the ratio of *X/Y:*

$$E = \frac{\%\ change\ in\ Y}{\%\ change\ in\ X} = \frac{\Delta Y/Y}{\Delta X/X} = \frac{\Delta Y}{\Delta X}\cdot\frac{X}{Y} = slope\cdot\frac{X}{Y}.$$

It is only for the double-log, or log-linear, model that the two are identical. Because of this special feature, the double-log or log linear model is also known as the constant elasticity model (since the regression line is a straight line in the logs of Y and X, its slope is constant throughout, and elasticity is also constant – it doesn't matter at what value of X this elasticity is computed).

2. THE SEMILOG MODEL (LOG-LIN)

Economists, businesspeople and the government are often interested in finding out the rate of growth of certain economic variables (if *Y(t)* and *Y(t-1)* are values of the variable at time *t* and *t-1*, then the rate of growth of *Y* between the two time periods is measured as

$$\frac{Y(t) - Y(t-1)}{Y(t-1)} \cdot 100,$$

which is simply the relative, or proportional, change in *Y* multiplied by 100).

To measure the growth rate over a longer period of time one can use a semilog regression model (semilog – because only one variable appears in the logarithmic form).

Suppose that we want to measure the rate of growth of consumer credit outstanding *(Y)* over a certain period:

$$Y(t)=Y_o\cdot(1+r)^{t}, \qquad (2.1)$$

where Y_o= the initial value of *Y*, *Y(t)*=*Y*'s value at time *t*, *r*=the compound (that is, over time) rate of growth of *Y*.

Let's take the natural log of equation (2.1) on both sides to obtain

$$lnY(t)=lnY_o+t\cdot ln(1+r). \qquad (2.2)$$

Now let $\alpha=lnY_o$ and $\beta=ln(1+r)$. Therefore we can express model (2.2) as

$$lnY(t)=\alpha+\beta\cdot t. \qquad (2.3)$$

In this model

$$\beta = \frac{\partial lnY(t)}{\partial t} = \frac{\Delta Y/Y}{\partial t} = \frac{\text{relative change in } Y}{\text{absolute change in } t}.$$

So in the semilog model like regression (2.3) the slope coefficient measures the relative change in Y for a given absolute change in the explanatory variable. If the relative change is multiplied by 100, we obtain the percentage change, or the growth rate.

3. THE SEMILOG MODEL (LIN-LOG)

Let's consider a model where the dependent variable is in the linear form but the explanatory variable is in the log form:

$$Y(t)=\alpha+\beta\cdot \ln X, \tag{3.1}$$

where Y=the GNP, X=the money supply.

$$\frac{\partial Y}{\partial X}=\beta\cdot\frac{1}{X}, \quad \text{therefore } \beta=X\cdot\frac{\partial Y}{\partial X}=\frac{\Delta Y}{\Delta X/X}=$$

$$=\frac{\textit{absolute change in Y}}{\textit{relative change in X}}. \tag{3.2}$$

$$\Delta Y=\beta\cdot\frac{\Delta X}{X}, \tag{3.3}$$

so the absolute change in Y $(=\Delta Y)$ is equal to β times the relative change in X. If the latter is multiplied by 100, then (3.3) gives the absolute change in Y for a percentage change in X. Thus, if X changes by 0.01 unit (or 1 percent), the absolute change in Y is $0.01\cdot\beta$.

If the money supply increases by 1 percent on the average the GNP increases by $\beta/100$ units.

NB!

There are more functional forms…

4. Reciprocal models

$Y(i)=\alpha+\beta\cdot(1/X(i))\cdot$

with two important applications – the Engel expenditure curve and the celebrated Phillips curve of macroeconomics;

the salient feature of these models is that as X increases indefinitely, the term 1/X(i) approaches zero and Y approaches the limiting value of α; therefore, models like this have built into them a limit value that the dependent variable will take when the value of X increases indefinitely.

5. Polynomial regression models

such as, for example, a cubic function relating total cost of production (Y) to the output (X)

$Y(i)=\alpha+\beta_1\cdot X(i)+\beta_2\cdot X(i)^2+\beta_3\cdot X^3(i);$

In this type of functions there is only one explanatory variable on the right-hand side, but it appears with various powers, thus making them multiple regression models.

6. Interaction Models

such as, for example,

$Y(i)=\alpha+\beta_1\cdot X_1(i)+\beta_2\cdot X_2(i)+\beta_3\cdot X_1(i)\cdot X_2(i)$

with the interaction term $X_1(i)\cdot X_2(i)$.

If the interaction term were omitted, the effect of X_1 *on Y would be measured by* β_1*. However, with the interaction, the effect is* $\beta_1+\beta_3\cdot X_2(i)$ *(we got this by differentiating with respect to* X_1*). Thus, the effect of* X_1 *on Y depends on the level of the variable* X_2*. If* β_3 *is positive, the effect of* X_1 *on Y will increase as the value of* X_2 *increases.*

For example, the effect of the education (X_1*) on the wages (Y) depends on the gender (male or female, (*X_2*)).*