

LECTURE NOTES II

FUNCTIONAL FORMS OF REGRESSION MODELS

The term “linear regression” means a regression that is *linear in the parameters* (that is, the parameters are raised to the power of 1 only), *LIP*; it may or may not be *linear in explanatory variables*, *LIV*.

1. THE LOG-LINEAR MODEL

Suppose we want to estimate the price elasticity of demand, that is, the percentage change in the quantity demanded for a percentage change in the price (of a given commodity). We cannot estimate it from the LIP/LIV model since the slope coefficient of that model simply gives the absolute change in the average quantity demanded for a unit change in the price of the commodity, but this is not elasticity. But it can be readily computed from the so called log-linear model:

$$\ln Y(t) = \alpha + \beta \cdot \ln X(t) \quad (1)$$

(ln=the natural logarithm, that is, logarithm to the base e).

Such models are called log-linear (because of linearity in the logs of the variables) or double-log (because both variables are in the log form).

By definition, elasticity is

$$E = \frac{\partial Y}{\partial X} \cdot \frac{X}{Y},$$

where $\partial Y / \partial X$ means the derivative of Y with respect to X.

In the double-log model (1)

$$\beta = \frac{\partial \ln Y}{\partial \ln X} = \frac{\Delta \ln Y}{\Delta \ln X} = \frac{\Delta Y/Y}{\Delta X/X} = \frac{\Delta Y}{\Delta X} \cdot \frac{X}{Y} = E$$

(since $\Delta \ln Y / \Delta \ln X$ is an approximation of $\partial \ln Y / \partial \ln X$ and a change in the log of a number is a relative or proportional change:

$$\Delta \ln Y = \Delta Y / Y, \Delta \ln X = \Delta X / X).$$

Thus, if Y represents the quantity of a commodity demanded and X represents its unit price, β measures the price elasticity of demand.

Actually, in general, elasticity is equal to the slope times the ratio of X/Y:

$$E = \frac{\% \text{ change in } Y}{\% \text{ change in } X} = \frac{\Delta Y / Y}{\Delta X / X} = \frac{\Delta Y}{\Delta X} \cdot \frac{X}{Y} = \text{slope} \cdot \frac{X}{Y}$$

It is only for the double-log, or log-linear, model that the two are identical. Because of this special feature, the double-log or log linear model is also known as the constant elasticity model (since the regression line is a straight line in the logs of Y and X, its slope is constant throughout, and elasticity is also constant – it doesn't matter at what value of X this elasticity is computed).

2. THE SEMILOG MODEL (LOG-LIN)

Economists, businesspeople and the government are often interested in finding out the rate of growth of certain economic variables (if $Y(t)$ and $Y(t-1)$ are values of the variable at time t and $t-1$, then the rate of growth of Y between the two time periods is measured as

$$\frac{Y(t) - Y(t-1)}{Y(t-1)} \cdot 100,$$

which is simply the relative, or proportional, change in Y multiplied by 100).

To measure the growth rate over a longer period of time one can use a semilog regression model (semilog – because only one variable appears in the logarithmic form).

Suppose that we want to measure the rate of growth of consumer credit outstanding (Y) over a certain period:

$$Y(t) = Y_o \cdot (1+r)^t, \quad (2.1)$$

where Y_o = the initial value of Y , $Y(t)$ = Y 's value at time t , r = the compound (that is, over time) rate of growth of Y .

Let's take the natural log of equation (2.1) on both sides to obtain

$$\ln Y(t) = \ln Y_o + t \cdot \ln(1+r). \quad (2.2)$$

Now let $\alpha = \ln Y_o$ and $\beta = \ln(1+r)$. Therefore we can express model (2.2) as

$$\ln Y(t) = \alpha + \beta \cdot t. \quad (2.3)$$

In this model

$$\beta = \frac{\partial \ln Y(t)}{\partial t} = \frac{\Delta Y/Y}{\partial t} = \frac{\text{relative change in } Y}{\text{absolute change in } t}.$$

So in the semilog model like regression (2.3) the slope coefficient measures the relative change in Y for a given absolute change in the explanatory variable. If the relative

change is multiplied by 100, we obtain the percentage change, or the growth rate.

3. THE SEMILOG MODEL (LIN-LOG)

Let's consider a model where the dependent variable is in the linear form but the explanatory variable is in the log form:

$$Y(t) = \alpha + \beta \cdot \ln X, \quad (3.1)$$

where Y =the GNP, X =the money supply.

$$\frac{\partial Y}{\partial X} = \beta \cdot \frac{1}{X}, \quad \text{therefore } \beta = X \cdot \frac{\partial Y}{\partial X} = \frac{\Delta Y}{\Delta X / X} =$$

$$\frac{\text{absolute change in } Y}{\text{relative change in } X}. \quad (3.2)$$

$$\Delta Y = \beta \cdot \frac{\Delta X}{X}, \quad (3.3)$$

so the absolute change in Y ($=\Delta Y$) is equal to β times the relative change in X . If the latter is multiplied by 100, then (3.3) gives the absolute change in Y for a percentage change in X . Thus, if X changes by 0.01 unit (or 1 percent), the absolute change in Y is $0.01 \cdot \beta$.

If the money supply increases by 1 percent on the average the GNP increases by $\beta/100$ units.

NB!

There are more functional forms...

4. Reciprocal models

$$Y(i) = \alpha + \beta \cdot (1/X(i))$$

with two important applications – the Engel expenditure curve and the celebrated Phillips curve of macroeconomics;

the salient feature of these models is that as X increases indefinitely, the term $1/X(i)$ approaches zero and Y approaches the limiting value of α ; therefore, models like this have built into them a limit value that the dependent variable will take when the value of X increases indefinitely.

5. Polynomial regression models

such as, for example, a cubic function relating total cost of production (Y) to the output (X)

$$Y(i) = \alpha + \beta_1 \cdot X(i) + \beta_2 \cdot X(i)^2 + \beta_3 \cdot X(i)^3;$$

In this type of functions there is only one explanatory variable on the right-hand side, but it appears with various powers, thus making them multiple regression models.

6. Interaction Models

such as, for example,

$$Y(i) = \alpha + \beta_1 \cdot X_1(i) + \beta_2 \cdot X_2(i) + \beta_3 \cdot X_1(i) \cdot X_2(i)$$

with the interaction term $X_1(i) \cdot X_2(i)$.

If the interaction term were omitted, the effect of X_1 on Y would be measured by β_1 . However, with the interaction, the effect is $\beta_1 + \beta_3 \cdot X_2(i)$ (we got this by differentiating with respect to X_1). Thus, the effect of X_1 on Y depends on the level of the variable X_2 . If β_3 is positive, the effect of X_1 on Y will increase as the value of X_2 increases.

For example, the effect of the education (X_1) on the wages (Y) depends on the gender (male or female, (X_2)).